Data Storage

Number Systems

Lecture Contents

- Decimal
- Octal
- Binary
- Hexadecimal

Decimal Notation

- Using ***place value*** and ten ***digits*** (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and a separator (the decimal point, “.”) to represent a number.

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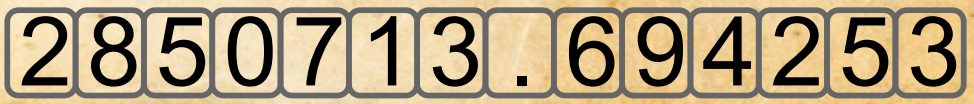
2850713.694253

Decimal Notation

- Using ***place value*** and ten ***digits*** (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and a separator (the decimal point, “.”) to represent a number.



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  - The above ***decimal system*** is only about 500 years old.
    - Ancient decimal systems did not express decimal ***fractions*** less than one
    - Even more ancient ***decimal systems*** did not use ***place value***; for example, ***Roman Numerals***, where a year such as 1998 is represented by:
      - MCMLXXXVIII

Decimal

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    - Even more ancient ***decimal systems*** did not use ***place value***; for example, ***Roman Numerals***, where a year such as 1998 is represented by:
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- More generally, a ***decimal system*** is a number system that uses ***base ten*** to represent a number (there are ten different symbols).

Place Value for Decimal Numbers

- Recall what you learned about *place value* for *decimal* numbers:

275

$5 \times 10^0 = 5$

$7 \times 10^1 = 70$

$2 \times 10^2 = 200$

Octal

Octal

- A number system that uses **base 8** (uses only 8 different symbols)

Octal

- A number system that uses **base 8**

Decimal	Octal
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Octal

- A number system that uses **base 8**

Decimal	Octal
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
⋮	

Octal

- A number system that uses **base 8**

Decimal	Octal
0	0
1	1
2	2
3	3
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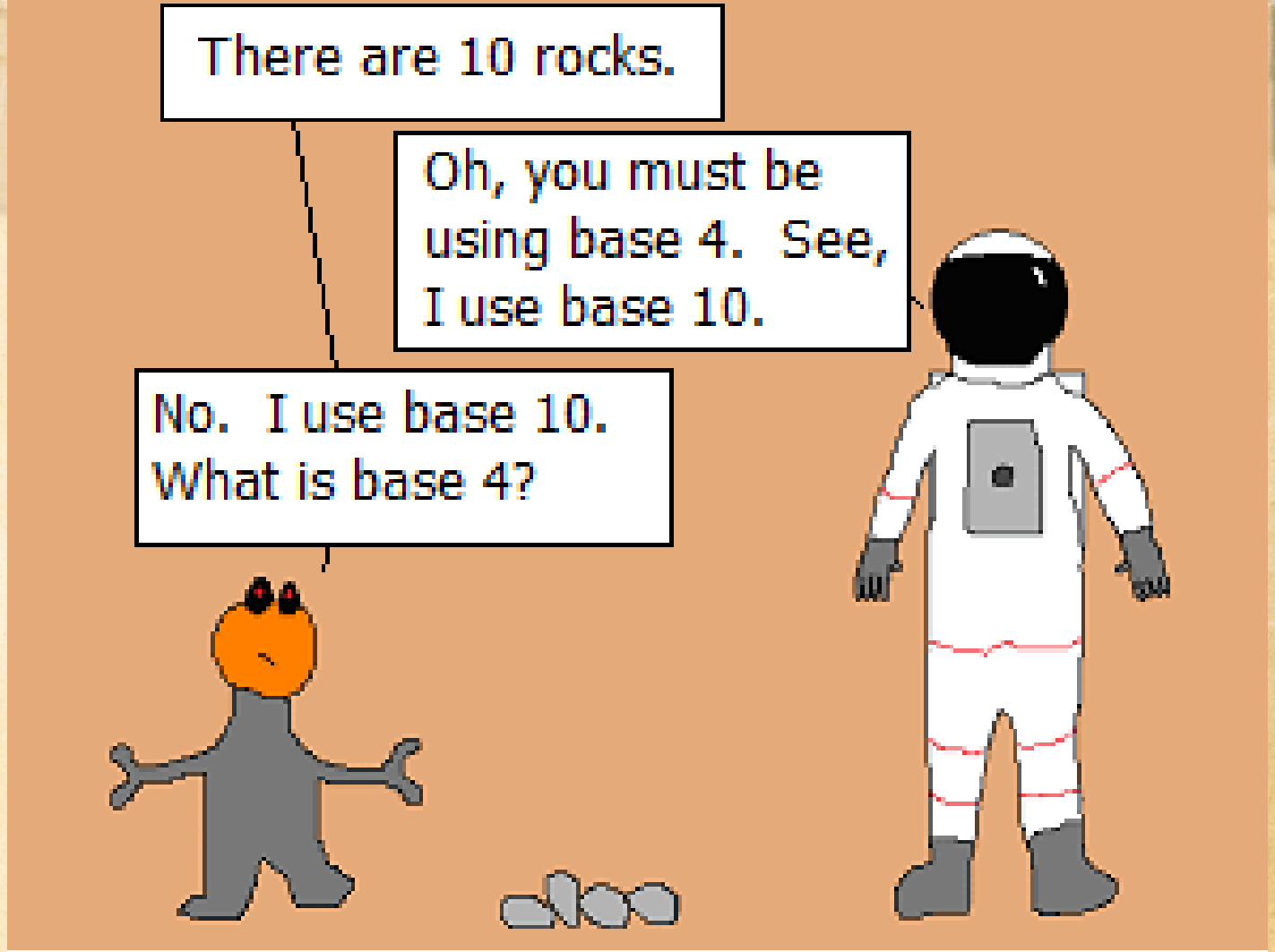
⋮	

Octal

- A number system that uses **base 8**

Decimal	Octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11
10	⋮
11	
⋮	

Octal



Every base is base 10.

Place Value for Octal Numbers

- Recall what you learned about *place value* for *decimal* numbers:

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Octal

$5 \times 8^0 = 5$

$7 \times 8^1 = 70$

$2 \times 8^2 = 200$

Decimal

5

56

128

189

Octal

- A number system that uses **base 8**

Decimal addition

$$\begin{array}{r} 347 \\ +\,39 \\ \hline \end{array}$$

Octal addition

$$\begin{array}{r} 533 \\ +\,47 \\ \hline \end{array}$$

Octal

- A number system that uses **base 8**

Decimal addition

$$\begin{array}{r} 347 \\ +39 \\ \hline 386 \end{array}$$

Octal addition

$$\begin{array}{r} 533 \\ +47 \\ \hline 602 \end{array}$$

Octal

- A number system that uses **base 8**
  - Octal Multiplication

$$\begin{array}{r} 27 \\ \times 12 \\ \hline \\ \\ \hline \end{array}$$

Octal Multiplication Table

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	2	3	4	5	6	7
2	2	4	6	10	12	14	16
3	3	6	11	14	17	22	25
4	4	10	14	20	24	30	34
5	5	12	17	24	31	36	43
6	6	14	22	30	36	44	52
7	7	16	25	34	43	52	61

Octal

- A number system that uses **base 8**
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$$\begin{array}{r} 27 \\ \times 12 \\ \hline 56 \\ 270 \\ \hline 346 \end{array}$$

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Binary

- A number system that uses **base 2** (uses only two different symbols)

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Decimal	Octal	Binary
0	0	0
1	1	1
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	10	
9	11	
10	⋮	
11		
⋮		

Binary

- A number system that uses **base 2** (uses only two different symbols)

Decimal	Octal	Binary
0	0	0
1	1	1
2	2	10
3	3	11
4	4	
5	5	
6	6	
7	7	
8	10	
9	11	
10	⋮	
11		
⋮		

Binary

- A number system that uses **base 2** (uses only two different symbols)

Decimal	Octal	Binary
0	0	0
1	1	1
2	2	10
3	3	11
4	4	100
5	5	101
6	6	110
7	7	111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
⋮	⋮	⋮

Binary

- Converting ***binary*** to ***decimal***

Binary

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  - Recall what you learned about *place value* for *decimal* numbers:

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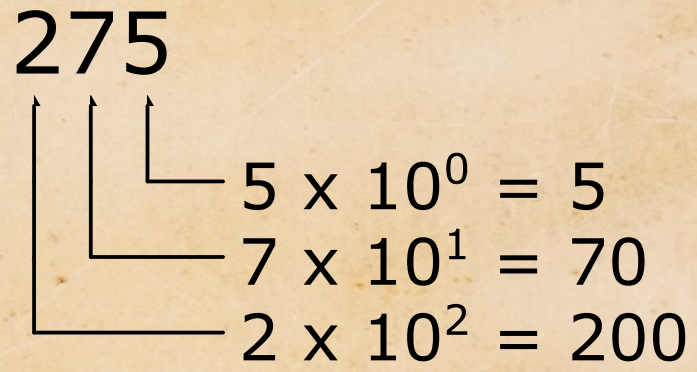
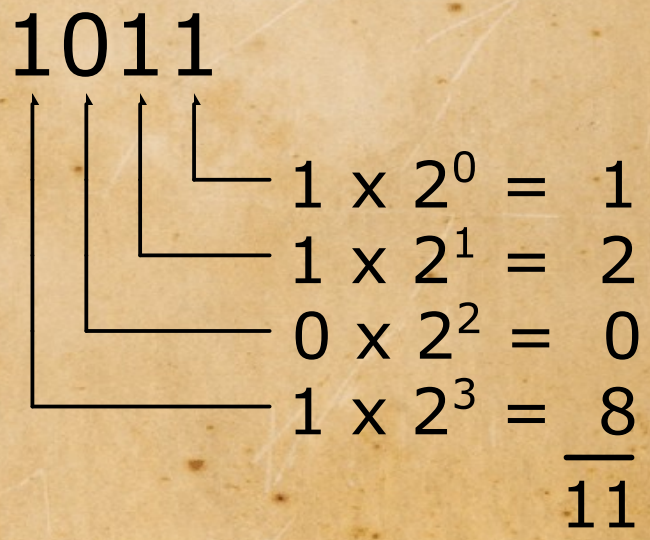
Binary

- Converting ***binary*** to ***decimal***
  - *Place value* is useful for converting binary to decimal

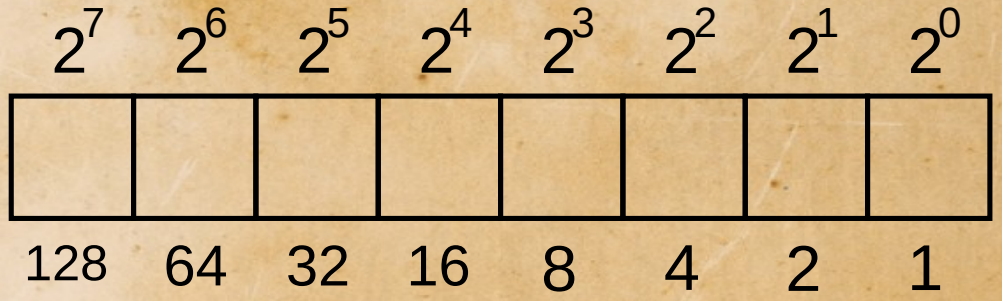




Binary

- Converting ***decimal*** to ***binary***
  - We use *place value* to convert 75 decimal into binary...

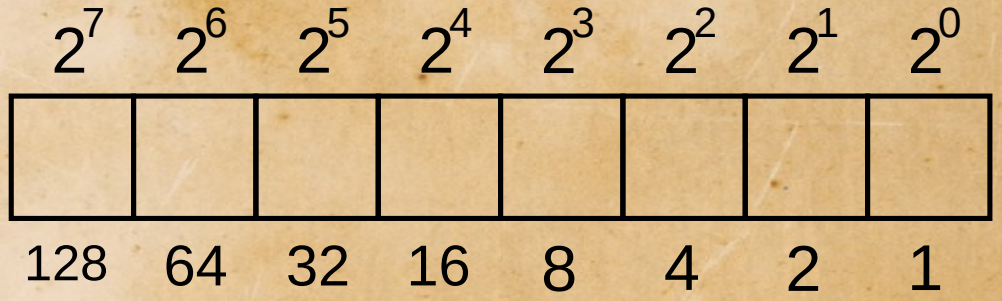
$$75 - 64 = 11$$



Binary

- Converting ***decimal*** to ***binary***
  - We use *place value* to convert 75 decimal into binary...

75−64 = 11
11−8 = 3
3−2 = 1
1−1 = 0



Binary

- Converting ***decimal*** to ***binary***
  - We use *place value* to convert 75 decimal into binary...

$75 - 64 = 11$

$11 - 8 = 3$

$3 - 2 = 1$

$1 - 1 = 0$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	0	1	0	1	1
128	64	32	16	8	4	2	1

Binary

- Converting ***decimal*** to ***binary***
  - Divide by 2 to convert 75 decimal into binary...

$$75 \div 2 = 37 \quad r\ 1$$

Binary

- Converting ***decimal*** to ***binary***
  - Divide by 2 to convert 75 decimal into binary...

$$75 \div 2 = 37 \quad r\ 1$$
$$37 \div 2 = 18 \quad r\ 1$$

Binary

- Converting ***decimal*** to ***binary***
  - Divide by 2 to convert 75 decimal into binary...

75 ÷ 2 = 37 r 1
37 ÷ 2 = 18 r 1
18 ÷ 2 = 9 r 0
9 ÷ 2 = 4 r 1
4 ÷ 2 = 2 r 0
2 ÷ 2 = 1 r 0
1 ÷ 2 = 0 r 1

Binary

There are 10 types of people
in this world.

Those who understand binary,
and those who don't.

Binary

- A number system that uses **base 2** (uses only two different symbols)
  - Binary addition

$$\begin{array}{r} 1011 \\ +1101 \\ \hline \end{array}$$

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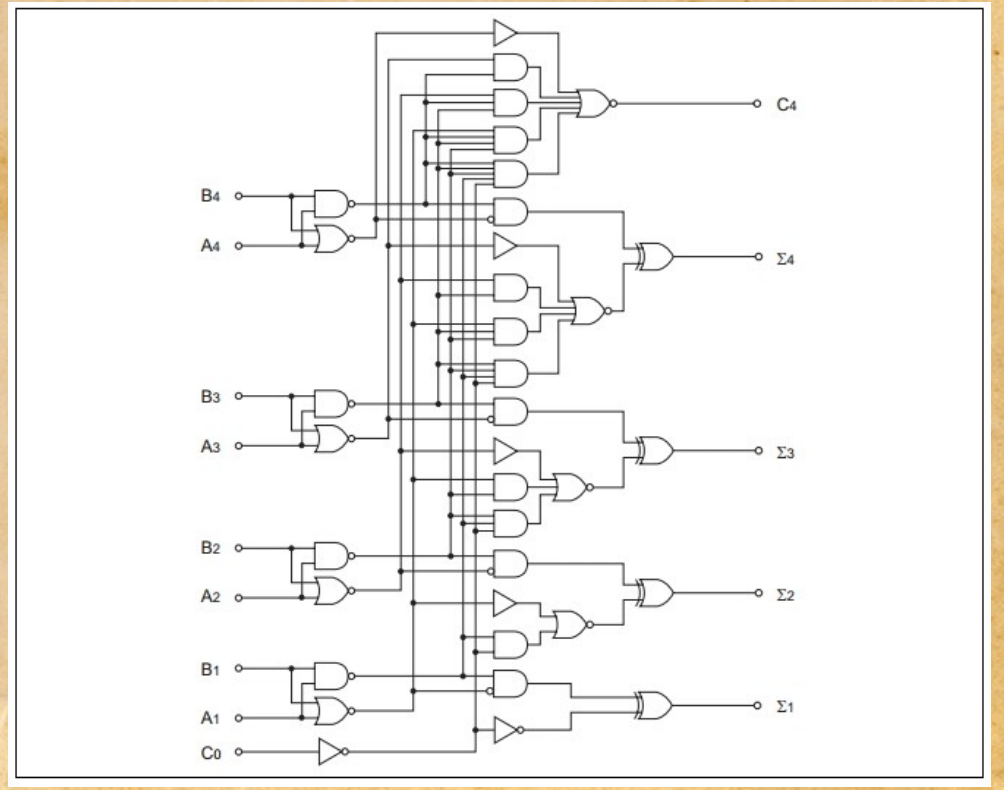

Binary

- A number system that uses **base 2** (uses only two different symbols)
  - Binary multiplication

$$
\begin{array}{r}
101 \\
\times 110 \\
\hline
000 \\
1010 \\
10100 \\
\hline
11110
\end{array}
$$

Binary

- A number system that uses **base 2** (uses only two different symbols)
  - Binary multiplication

101	5
×110	×6
000	
1010	
10100	
11110	30

Binary

- An interesting thing about dividing or multiplying by 2 for a binary number…
  - For a decimal number, multiplying by 10

$$13 \times 10 = 130$$

    - We shift to the left, putting a zero in the ones place.

Binary

- An interesting thing about dividing or multiplying by `2` for a binary number…
  - For a decimal number, multiplying by 10

$$13 \times 10 = 130$$

    - We shift to the left, putting a zero in the ones place.
  - For a binary number, multiplying by 2:

$$1011 \times 10 = 10110$$

    - We shift to the left, putting a zero in the ones place
    - Note: `10` in binary is `2` in decimal.
    - In Java, the bitwise shift operators multiply “ `<<` ” and divide “ `>>` ” by powers of 2.

Hexadecimal

- A number system that uses **base 16**
  - Why is it used?

Hexadecimal

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    - More human-readable than ***binary***
    - Very easy to convert between ***binary*** and ***hexadecimal***

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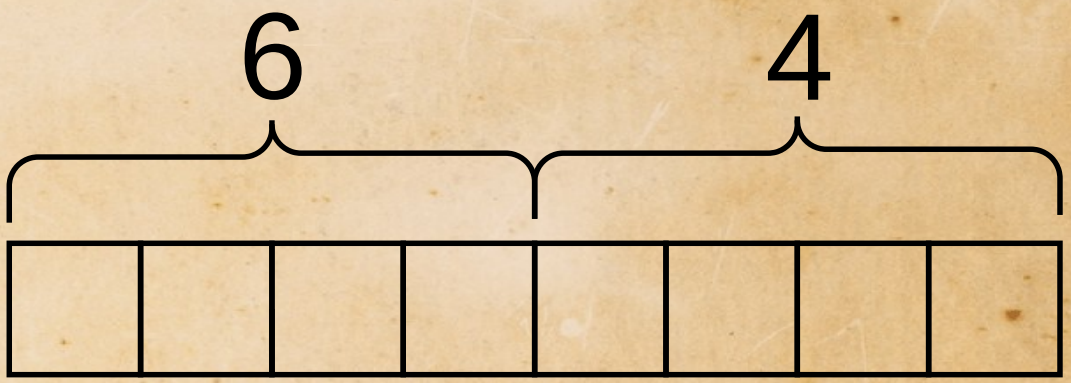

Decimal	Hexa-decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hexadecimal

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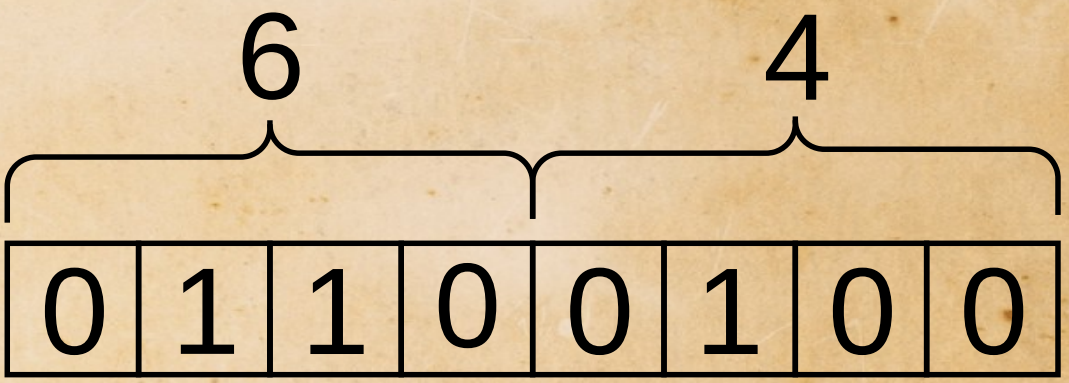

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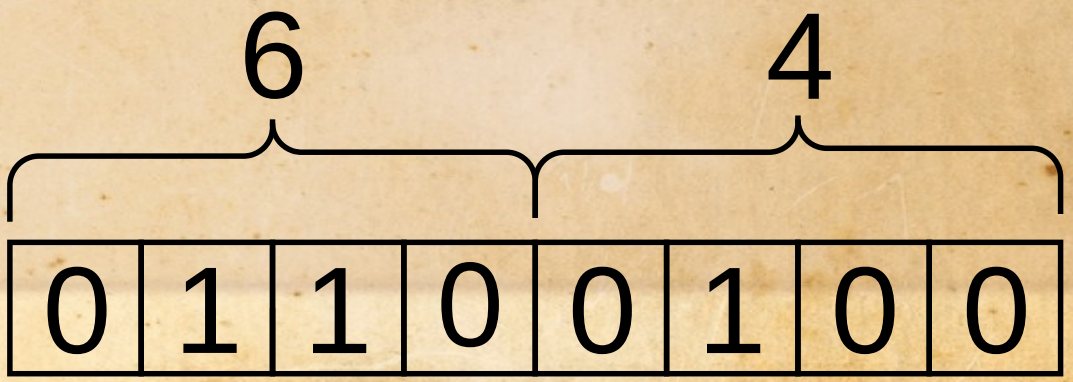
Hexadecimal

- Important note: in computer science, *hexadecimal* numbers are usually prefixed with `0x`
  - Decimal 12 = `0xC`
  - Decimal 254 = `0xFE`
  - Decimal 100 = `0x64`



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11	B	1011
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13	D	1101
14	E	1110
15	F	1111

ASCII Character Codes (in hexadecimal)

Hexadecimal	Char	Hexadecimal	Char
41	A	61	a
42	B	62	b
43	C	63	c
44	D	64	d
45	E	65	e
46	F	66	f
47	G	67	g
48	H	68	h
49	I	69	i
4A	J	6A	j
4B	K	6B	k
4C	L	6C	l
4D	M	6D	m
4E	N	6E	n
4F	O	6F	o
50	P	70	p
51	Q	71	q
52	R	72	r
53	S	73	s
54	T	74	t
55	U	75	u
56	V	76	v
57	W	77	w
58	X	78	x
59	Y	79	y
5A	Z	7A	z

Data Storage

Number Systems